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REGULAR ARTICLE

Rationally mispriced assets in equilibrium

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Abstract We present a simple model of trading in a financial market where agents are asymmetrically informed and information is transmitted through the price system. We characterize the equilibrium for this economy and show that 'rational mispricing' of assets occurs if the price system fails to reveal the insider information accurately. It is argued that the communication of wrong information through equilibrium prices is compatible with full rationality on the part of the investors and may explain deviations from the efficient markets hypothesis.

Keywords Asymmetric information · Rational expectations · Mispricing · Efficiency

JEL Classification D43 · D51 · D8

Introduction

Modern finance theory is cast in the tradition of the neoclassical paradigm that investors make rational choices based on rational expectations. Specifically, agents are assumed to be unbiased Bayesian forecasters. As a description of economic behavior, this assumption is sometimes criticized on the grounds that it not only takes the complexity of the individual decision making process to the limits of acceptability, but is also descriptively false and incomplete (Akerlof and Dickens 1982; De Bondt and Thaler 1994). Critics point to recent findings

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in empirical finance which suggest that stock returns are much more predictable than can be rationalized in the presence of an efficient market mechanism.

According to the Efficient Capital Markets Hypothesis,[1] all publicly available information is at any time reflected in the asset prices. Yet, it has long been suggested that the empirically observed risk premia on the capital market and the erratic movements of stock prices are incompatible with efficient asset valuation and rational investor behavior [e.g., mean reversion in stock returns (De Bondt and Thaler 1989), the 'equity premium puzzle' of Mehra and Prescott (1985), the 'excess volatility puzzle' of Shiller (1981) and LeRoy and Porter (1981)]. [2] Moreover, stock market crashes are very difficult to reconcile with the Efficient Markets Hypothesis. The information publicized just prior to stock market crashes does not usually warrant the huge drop in stock prices (see Cutler et al. 1989). Quite often, the only financial news on the day of a stock market crash is the crash itself.

In order to understand these valuation anomalies in finance that appear puzzling within the standard paradigm, theorists have constructed models that allow for rational expectations equilibria in which private information is not fully revealed. These models use various techniques in order to incorporate additional sources of uncertainty. For example, the noise trader approach[3] postulates the existence of two kinds of agents, some fully rational (professional investors or arbitrageurs) and some less so (so-called noise traders or liquidity traders) (see e.g., Campbell and Kyle 1993; DeLong et al. 1990; Russell and Thaler 1985). The noise traders are assumed to trade randomly, and they have lower expected utility than rational agents although, due to unintentional risk bearing, in some circumstances they may earn more than the market return. An alternative approach assumes that aggregate supply in the market is random. In addition, agents only observe their own endowments while aggregate supply is unobservable and, therefore, constitutes a noise variable (e.g., Diamond and Verecchia 1981). A further strand of literature is based on the behavioral assumption that, due to psychological factors, agents are overconfident about the precision of their private information (Daniel et al. 1998; Odean 1998). While models based on quasirational investor behavior have met with some success in explaining the empirical asset valuation anomalies, it is hard to understand the larger environment which would motivate quasirational agents to participate in the market.

In this paper we follow a different line. Our approach introduces extra noise via the uncertainty about the presence of private information in the market. This type of uncertainty has been used in various microstructure models (see, for instance, Easley and O'Hara 1991, 1992; Easley et al. 1998). In these models

1 For an overview see Malkiel 2003 or Bossaerts 2002.

2 In a series of recent papers Drees and Eckwert (1995, 1997, 2000) suggest as an explanation for the poor empirical performance of theoretical asset pricing models that the maintained specification of preferences is too rigid. Jackson and Peck (1999) analyze whether asymmetric information could be a possible cause of excess asset price volatility.

3 See Shiller (2003) for a survey related to this field.

of partial equilibrium insiders are assumed to be risk neutral and the probabilistic structure is described by means of discrete random variables which assume only two values (high and low). Our paper, by contrast, uses a general equilibrium setting where risk averse agents interact on competitive markets. We keep the framework simple in order to avoid that severe restrictions need to be placed on the stochastic structure of the economy.

The main goal of this study is to show that full rationality on the part of the investors does not rule out over- and undervaluation of stocks and other assets in equilibrium. We will call these deviations from the Efficient Markets Hypothesis 'rational mispricing of assets'. Mispricing occurs because some agents rationally draw wrong conclusions from the price system about the information received by other (better informed) investors. Under specific conditions market prices not only fail to aggregate all relevant information about market fundamentals, but they send out wrong information signals. As a consequence, some agents rationally base their decisions on 'distorted' information sets, thereby adding a substantial distortionary component to the asset price. This mechanism drives a wedge between the 'efficient' price, which is the market clearing price when no wrong information is added to the individual information sets, and the actual market price which is distorted by rational informational errors. A model is developed below that contains these elements in as simple a form as possible. Despite some similarities in the information structure, our approach contrasts sharply with the Easley and O'Hara models where, due to a narrower equilibrium concept, rational misinterpretations of the observed price system cannot occur.

Our paper analyzes mispricing phenomena in a perfectly rational setting. Similar to much of the literature on banking panics or financial crises, the paper deals with zero probability events. A theoretical setup, which assigns zero probability to some events under investigation, does not imply that these events will never occur. Rather it should be regarded as a methodology to formalize the idea that the analysis focuses on *unexpected* events which have not been anticipated by the agents ex ante.

We start with a simple model that includes differentially informed investors. The modeling draws from the pioneering works by Bray (1981), Grossman and Stiglitz (1980), and Hellwig (1980). However, closest to our analysis are the works by Grundy and McNichols (1989), Mirman and Reisman (1988), and Romer (1993), which all offer rational interpretations of price movements without the arrival of outside news. The papers by Grundy and McNichols and by Mirman and Reisman appeal to self-fulfilling expectations as the central mechanism through which rational reassessments of asset fundamentals occur in the absence of new information. However, unlike in our approach, in equilibrium no wrong information is transmitted by the price system and consequently no 'mispricing' occurs. The work by Romer models an economy in which the trading process itself reveals some information about the investors' assessments of asset fundamentals during periods in which the market does not receive any news that could plausibly be the source of changes in asset prices. Critical to his analysis is the assumption that the stock market does an imperfect job of revealing the information possessed by asymmetrically informed investors.

In this paper we analyze an economy which extends over two time periods. There exist two types of agents, 'informed' and 'uninformed'. The informed traders observe with some probability $\alpha \in (0,1)$ an information signal which allows them to calculate the true underlying probability distribution of future asset payoffs. Thus, there is a positive probability of $(1-\alpha)$ (possibly close to zero) that the market does not receive any outside news. Uninformed traders know that the information of the informed agents is (partially) reflected in the current price system, so they form their beliefs about future asset payoffs on the basis of the information which they can learn from observing current prices. However, as there are two sources of information (the knowledge about the presence of insider information and the value of the insider signal itself) but only one asset price, the price system fails to perfectly reveal the private information.[4] We characterize the equilibrium price function and show that it exhibits a discontinuity for some critical realization of the information signal. This discontinuity arises because the uninformed traders misinterpret the price system. They conclude (rationally but erroneously) from observing current prices that the market has not yet received any private information. Based on this belief they take a position in the market.

Depending on the parameters of the model the assets can be rationally overpriced or rationally underpriced at this point of discontinuity. Moreover, the critical value of the realization (and, in fact, the whole equilibrium price function) is independent of the probability α that the signal will be observed by the informed traders. There is another value of the information signal which is of critical importance. For this realization no market clearing price exists: the 'efficient' asset price induces the uninformed agents to make inaccurate inferences about the information signal, while any other price reveals the information. Thus neither price equilibrates the market.

From an empirical perspective, our results have a number of testable implications. One prediction of our model is that strong price fluctuations which occur at times when hardly any new information becomes known go hand in hand with large changes in trading volumes. A further, and perhaps more surprising prediction is that markets where private information becomes available with high frequency exhibit the same pattern of mispricing as markets where this frequency is low.

The plan of the paper is as follows: 'The model' section outlines the model and derives the optimal portfolio decisions of the asymmetrically informed agents. 'The equilibrium' section studies the properties of the rational expectations equilibrium, and shows how the transmission of information through the price system can result in a mispricing of assets. Finally, we offer a few concluding comments in 'A numerical example'. Some technicalities are relegated to a separate Appendix.

[4] For a more general treatment of the conditions under which a price system fully reveals private information see e.g., Ausubel (1990) or Allen (1986).

The model

Consider a two-period model with one consumption good. There are two types of traders who differ by their endowments and preferences as well as by their information structures. There also exists one (risky) asset, which can be used to transfer wealth across periods. In period 0 the risky asset can be traded on a competitive stock market. In period 1 the asset yields a (stochastic) real dividend

$$\tilde{d} = \tilde{\epsilon} + \tilde{y}, \tag{1}$$

where $\tilde{\epsilon}$ and $\tilde{y}$ are stochastically independent random variables. $\tilde{\epsilon}$ denotes an information signal (with generic realization ϵ), which realizes in period 0 and takes values in $K \subset \mathbb{R}$. With probability α the signal 'reaches' the market in which case it will be observed by one class of traders, the insiders. The other class of traders, the outsiders, do not know whether or not the market has received the outside news $\tilde{\epsilon}$, but they will try to infer this information from the price system. $\tilde{y}$ denotes noise which cannot be observed until period 1. The distribution of $\tilde{\epsilon}$ and $\tilde{y}$ is known to all market participants. It is assumed that both variables follow a continuous distribution with existing second moments:

$$E[\tilde{\epsilon}] = \mu_\epsilon > 0, \quad Var[\tilde{\epsilon}] = \sigma_\epsilon^2 > 0, \quad E[\tilde{y}] = 0, \quad Var[\tilde{y}] = \sigma_y^2 > 0. \tag{2}$$

Initially, all agents are endowed with one share of the asset. In period 0 the insiders face the following budget constraint:

$$px + c_0 = W, \tag{3}$$

where p denotes the period 0 price of the asset, x is the number of additional shares purchased, c_0 is consumption in period 0, and W denotes the insiders' goods endowment in period 0. Period 1 consumption equals the return on the agent's portfolio:

$$\tilde{c_1} = \tilde{d}(x+1). \tag{4}$$

It is assumed that the insiders' utility function is additively time separable:

$$E[U(c_0, \tilde{c_1})|\epsilon, p] := u_0(c_0) + E[u_1(\tilde{c_1})|\epsilon], \tag{5}$$

if they receive the signal, or

$$E[U(c_0, \tilde{c_1})|p] := u_0(c_0) + E[u_1(\tilde{c_1})], \tag{6}$$

if not, respectively. For convenience, the insiders decide upon a linear period 0 utility function

$$u_0(c_0) := bc_0 = b(W - px), \quad b > 0 \tag{7}$$

and have simple mean-variance preferences in period 1:

$$\begin{aligned} E[u_1(\tilde{c_1})|\epsilon] : &= E[\tilde{c_1}|\epsilon] - \frac{a}{2}\mathrm{Var}[\tilde{c_1}|\epsilon] \\ &= E[\tilde{d}(x+1)|\epsilon] - \frac{a}{2}\mathrm{Var}[\tilde{d}(x+1)|\epsilon] \\ &= (x+1)\epsilon - \frac{a}{2}(x+1)^2\sigma_y^2, \end{aligned} \tag{8}$$

if they receive the signal, or

$$\begin{aligned} E[u_1(\tilde{c_1})] : &= E[\tilde{c_1}] - \frac{a}{2}\mathrm{Var}[\tilde{c_1}] \\ &= (x+1)\mu_\epsilon - \frac{a}{2}(x+1)^2(\sigma_y^2 + \sigma_\epsilon^2), \end{aligned} \tag{9}$$

if not, respectively. $a > 0$ is a parameter measuring period 1 risk aversion. Utility maximization yields the following insider demand functions:

$$x = \frac{\epsilon - bp}{a\sigma_y^2} - 1, \tag{10}$$

if a signal is received, or

$$x = \frac{\mu_\epsilon - bp}{a(\sigma_y^2 + \sigma_\epsilon^2)} - 1, \tag{11}$$

if not, respectively. The preferences and endowments of the outsiders are symmetric to those of the insiders. In particular, the outsiders have mean-variance preferences, too. We distinguish variables and parameters pertaining to outsiders by an upper bar, i.e., $\bar{W}$ denotes outsiders' initial wealth, $\bar{a}$ is the outsiders' risk aversion parameter, and $\bar{b}$ is the marginal utility in period 0. Thus, essentially, both groups of agents differ only by their information sets. Since outsiders do not observe the signal ϵ, they update their beliefs (in a Bayesian way) on the basis of the information revealed to them by the current asset price. The expected period 1 utility of outsiders can be stated as

$$E[u_1(\tilde{c_1})|p] = E[\tilde{c_1}|p] - \frac{\bar{a}}{2}\mathrm{Var}[\tilde{c_1}|p].$$

We will see later that the equilibrium price either reveals the information or is completely uninformative. Thus, it is sufficient here to consider the two cases in which prices reveal either all or no information. The same procedure as above then yields the demand function $\bar{x}(\cdot)$ for outsiders, once their information set is determined. Straightforward calculation shows that

$$\bar{x} = \frac{\epsilon - \bar{b}p}{\bar{a}\sigma_y^2} - 1, \tag{12}$$

if the outsiders are able to extract the information ϵ from the price p, and

$$\bar{x} = \frac{\mu_\epsilon - \bar{b}p}{\bar{a}(\sigma_y^2 + \sigma_\epsilon^2)} - 1, \tag{13}$$

if the outsiders rationally conclude that the signal has not been observed and hence the price does not contain any information about $\tilde{\epsilon}$.

The equilibrium

Let $0 < \lambda < 1$ be the share of insiders in the economy. For convenience, assume $a = \bar{a}$. We are looking for an equilibrium in which agents form rational expectations. Individual expectations are said to be rational, if they are based on both the agent's private information *and* the information revealed by the current asset price, and if the resulting individual asset demand functions lead precisely to this price.

Definition 1. *An* equilibrium *is a tuple (ϕ, φ) consisting of a set $\phi \subset \mathbb{R}$ and a correspondence φ between the signal space and the price space with the following properties:*

- *i) Any $p \in \phi$ clears the asset market when the insiders do not receive the signal $\tilde{\epsilon}$.*
- *ii) Any $p \in \varphi(\epsilon) := \{p : (\epsilon, p) \in \varphi\}$ equilibrates supply and demand on the asset market, when the insiders observe the realization ϵ of the signal.*
- *iii) The market participants act in a Bayesian sense meaning maximization of expected utility given their own observations (i.e., all traders observe p and insiders additionally observe ϵ, if available).*

Depending on the realized signal ϵ, the set of market clearing prices, $\varphi(\epsilon)$, may be empty, single-valued or multivalued, as the following proposition shows.

Proposition 1. *Define $\bar{p} := \frac{\mu_\epsilon - a(\sigma_y^2 + \sigma_\epsilon^2)}{\lambda b + (1-\lambda)\bar{b}}$ and assume $\bar{p} > 0$.*[5] *Then there exist constants $A > 0$, $\bar{A} > 0$, B and $\bar{B}$, such that $(\{\bar{p}\}, \varphi)$ is an equilibrium, where φ has the following structure:*

[5] $\bar{p}$ is the 'no information equilibrium price'. Assuming $\bar{p} > 0$ simplifies some technicalities without qualitatively affecting the analysis in the paper.

i) If $b \neq \bar{b}$, then the equilibrium price correspondence is of the form:

$$\varphi(\epsilon) = \begin{cases} \{A\epsilon + B\}, & \hat{\hat{\epsilon}} \neq \epsilon \neq \hat{\epsilon} \\ \{\bar{p}, A\epsilon + B\}, & \epsilon = \hat{\epsilon} \\ \emptyset, & \epsilon = \hat{\hat{\epsilon}} \end{cases}$$

ii) If $b = \bar{b}$, then the equilibrium price correspondence is of the form $\varphi(\epsilon) = \{A\epsilon + B\}$. In particular, φ is single-valued and fully revealing (in the sense of Allen 1986).

The following abbreviations are used:

$$\hat{\hat{\epsilon}} := A^{-1}(\bar{p} - B),\ \hat{\epsilon} := \bar{A}^{-1}(\bar{p} - \bar{B}). \tag{14}$$

Proof. See Appendix. □

We will call the equilibrium price $p \in \varphi(\epsilon)$ 'efficient', if it does not send out a wrong information signal to uninformed agents. More precisely, an equilibrium price is efficient, if no investor rationally draws wrong conclusions about the realization of the information signal ϵ from that price. The asset is rationally overpriced (underpriced), if the actual equilibrium price is higher (lower) than the efficient price. When all agents have the same preference structure (case (ii)), the equilibrium is fully revealing, because the price is monotonically increasing in the underlying market signal. The outsiders are always able to extract the market information out of the market price through $\epsilon = A^{-1}(p - B)$. In this case equilibrium can be described by a linear function. If the preference structure is heterogeneous (different marginal utilities in period 0), the situation is more complex (Fig. 1, drawn for $b < \bar{b}$, $A > \bar{A}$): the equilibrium price correspondence is no longer single-valued, but exhibits multi-valuedness as well as empty-valuedness at certain realizations of the information signal $\tilde{\epsilon}$. Any price $p \neq \bar{p}$ reveals the information through $\epsilon = A^{-1}(p - B)$ and is therefore efficient.

There are two possible equilibrium prices associated with the market information $\hat{\epsilon}$. $\hat{p} = A\hat{\epsilon} + B$ is the efficient equilibrium price which reveals the information $\hat{\epsilon}$. $\bar{p}$ is also a market clearing price, but one which leads uninformed investors astray and hence is not efficient. As shown in the proof, $\bar{p}$ is the market clearing price which comes up in a situation where there is no insider information available. This price is common knowledge and occurs with positive probability $1 - \alpha$. Thus, if the outsiders observe $\bar{p}$, they infer erroneously but completely rationally that the signal has not reached the market. This is a rational conjecture, because the probability that a signal (observed by the insiders) has led to the price $\bar{p}$ is zero,[6] while the event that no information arrived on the market has positive probability.

Thus $(\hat{\epsilon}, \bar{p})$ describes a situation, where the outsiders behave as if there were no information available – erroneously but completely rationally – although

[6] Note that the signal is drawn from a continuous distribution.

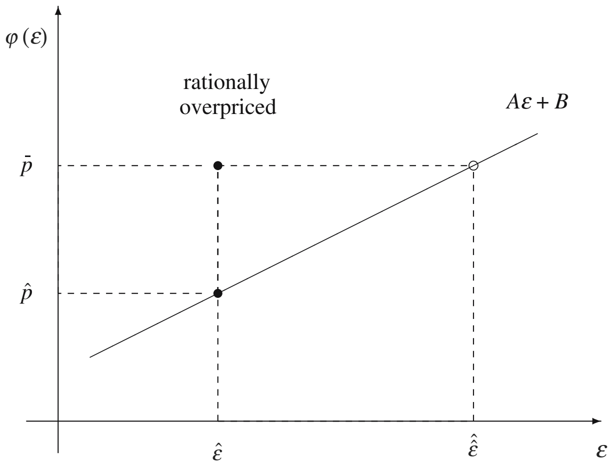

Fig. 1 Rationally overpriced asset

in fact there is and the equilibrium price does not reveal it. The discontinuity of the asset price is caused by a failure of the asset market to truthfully communicate information. Transmission of information through the price system would be perfect, if there were no uncertainty about the presence of insider information on the part of the outsiders. Thus, in our model, this uncertainty alone (characterized by the probability α) is the destabilizing force. One might therefore expect that the extent of the crash $|\hat{p} - \bar{p}|$ is sensitive to the value of α (the probability that there is a signal). This intuition is wrong, as demonstrated by our next proposition: as long as α lies strictly between zero and one, the gap between $\hat{p}$ and $\bar{p}$ is independent of the value of α. Thus, even the slightest amount of uncertainty about the presence of insider information may cause a significant dip of the stock market.[7]

Proposition 2. *The gap between $\hat{p}$ and $\bar{p}$ is given by*

$$\hat{p} - \bar{p} = \frac{1-\lambda}{(\lambda b + (1-\lambda)\bar{b})^2}(b - \bar{b})\sigma_\epsilon^2 \left(\frac{\mu_\epsilon}{\sigma_y^2 + \sigma_\epsilon^2} - a \right). \tag{15}$$

Proof. Follows from the definition of $\hat{p}$ and $\bar{p}$. □

It is easy to show that this gap is positive for $b > \bar{b}$ and negative for $b < \bar{b}$. In the case $b > \bar{b}$ ($\Rightarrow \hat{p} > \bar{p}$) a situation with signal $\hat{\epsilon}$ and market price $\bar{p}$ is possible, where the asset is *rationally underpriced*. Thus, in that case the

[7] There is another point of importance. Note that there is no equilibrium price associated with $\hat{\hat{\epsilon}}$. This market signal is not compatible with any price: the associated 'efficient' price induces the uninformed agents to draw wrong inferences from the price and so the equilibrium price does not lie on the line $A\epsilon + B$. Any other price in turn reveals the information, so that market clearing occurs only at the efficient price $A\hat{\hat{\epsilon}} + B$ (see the proof for details).

wrong information transmitted by the price $\bar{p}$ results in a lower asset value. Equivalently in the case $b < \bar{b}$ *rational overpricing* is possible. The important point is that this gap is independent of α, i.e., the mispricing phenomenon is robust in the sense that it does not disappear, if the probability $\alpha \in (0, 1)$ tends to 0 or 1. Thus, the full amount of mispricing occurs as soon as there is the slightest uncertainty about the presence of insider information on the market.

Corollary 1. *$|\hat{p} - \bar{p}|$ is increasing in μ_ϵ, σ_ϵ^2, and decreasing in a and σ_y^2.*

The amount of (possible) mispricing varies positively with the expected dividend of the underlying asset and increases as the signal becomes more volatile, whereas more volatility of noise and higher risk aversion on the part of the traders bring the informationally distorted price closer to its efficient value. The effect of an increasing λ is ambiguous. One can show that for $\bar{b} > b$ the gap $|\hat{p} - \bar{p}|$ depends positively on λ, just as in the case $\bar{b} < b$ and $\lambda > \bar{b}/(b - \bar{b})$. Restrictions on insider trading (decreasing λ) would dampen the extent of mispricing in that case. But this is not a general result in this model.

According to (10) and (11) the asset demand of an insider is strictly monotone decreasing in the asset price p. Thus, if the asset is rationally overpriced then the insiders sell more shares than they would do if the asset were priced efficiently. Conversely, if the asset is rationally underpriced then the insiders buy additional shares from the outsiders. The outsiders are willing to take the corresponding position on the other side of the market (i.e., to buy overpriced assets or to sell underpriced assets) because they misinterpret the asset price and therefore rationally base their decisions on false information. Although we have not conducted an explicit welfare analysis in this paper, our findings clearly suggest that the outsiders lose and the insiders benefit if the price system transmits inaccurate information.

We finally consider some empirical implications of our model. If rational mispricing occurs for some signal $\hat{\epsilon}$ then the price reacts extremely sensitive with regard to small changes in information at $\epsilon = \hat{\epsilon}$. Our model therefore predicts, as a testable empirical implication, that strong price fluctuations occurring at times when little new information becomes known should be accompanied by large changes in trading volumes. For example, when rational overpricing takes place then informed traders sell more assets and uninformed traders buy more assets than they would do otherwise. Symmetrically, rational underpricing leads to higher purchases by informed traders and higher sales by uninformed traders than would occur if assets were priced correctly. A further prediction of our model is that the amount of mispricing at $\epsilon = \hat{\epsilon}$ does not depend on the probability of private information being available in the market. Markets where private information becomes available over time with high frequency should, therefore, exhibit similar mispricing patterns as markets where this frequency is low. This implication might be subjected to empirical tests, since frequencies of private information flows typically vary widely across markets.

A numerical example

Above we have constructed an analytical framework in which rational mispricing occurs on a set of measure zero. To the extent that the mispricing phenomenon can be interpreted as a stock market crash, the model explains why such crashes are very rare events on real-life financial markets. While the continuous probabilistic specification buys us much analytical simplicity, it is worth pointing out that our main results, i.e., the occurrences of nonexistence and multiple equilibria, are no artifacts of models with a continuous state space in which these phenomena can only occur with zero probability. To demonstrate this, we numerically calculated a discrete-state version of our model, in which the state space consists of a finite number of possible events. Specifically, the market signal $\tilde{\epsilon}$ and the noise variable $\tilde{y}$ are uniformly distributed over the sets $\{1, 2, \ldots, 100\}$ and $\{-1, 0, 1\}$, respectively. Since the random variable $\tilde{\epsilon}$ has a high variance $((100^2 - 1)/12 = 833.25)$ relative to its mean $(101/2 = 50.5)$ we have chosen a low degree of risk aversion, $a = 0.001$, in order to balance the trade-off between risk and expected return in the agents' preferences. The total mass of insiders is 1/4 and the period 0 utilities are determined by $b = 10$ and $\bar{b} = 15$. $\alpha = 0.6$ is the probability that the information signal reaches the market.

The results of our calculation are reported in Fig. 2. Critical values of the market signal are $\hat{\epsilon} = 37$ and $\hat{\hat{\epsilon}} = 47$, both of which occur with probability 1/100. The prices $\bar{p} = 3.42$ and $p^{\text{eff}}(\hat{\epsilon}) = 2.69$ are both consistent with the

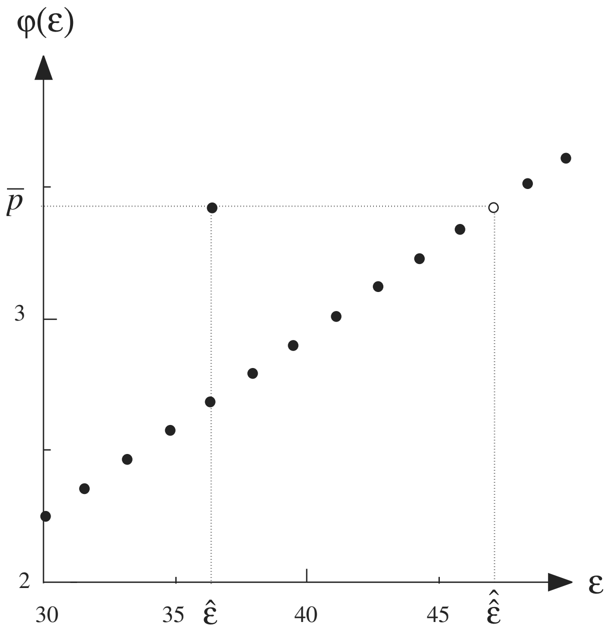

Fig. 2 A numerical example

realization $\hat{\epsilon}$, while no equilibrium price exists if the market receives the signal $\hat{\hat{\epsilon}}$. This implies that the relative amount of possible mispricing at $\epsilon = \hat{\epsilon}$ in this case is $(\hat{p} - \bar{p})/\hat{p} \approx 0,27$. While our numerical example does not produce any new qualitative results, it conveniently illustrates that the mispricing phenomenon discussed in this study is not limited to economies with atomless state spaces.

Conclusion

Financial markets are assumed by many economists to be the best real world example of the frictionless textbook model of perfect competition. In particular, informational problems are thought to be absent as long as the asset markets are competitive and traders have rational expectations: in such a world prices are likely to reflect all relevant private information about the value of the asset.

This study has shown that the informational efficiency of the price system breaks down if there is some uncertainty about the arrival of new information. Although the asset market is competitive, the price system may send out wrong information signals, thereby inducing some agents to rationally base their investment decisions on distorted information sets. As a consequence, assets are not valued efficiently. If rational mispricing occurs, then the insiders either sell overpriced assets to the outsiders or they buy underpriced assets from them. In both cases the outsiders lose and the insiders benefit from the price distortion. This approach not only gives a well-defined meaning to the notion of overvaluation and undervaluation of assets, but may also explain some of the asset valuation anomalies found in the empirical literature.

Due to the continuous framework we have chosen, the model generates rationally distorted asset prices as a rare (probability zero) event. While a continuous probabilistic specification buys us much analytical simplicity, it is worth pointing out that our main result, the occurrence of multiple equilibria at certain signals, is no artifact of models with a continuous state space. The general shape of the pricing function in Fig. 1 can be reproduced in a discrete environment as well, where all events occur with positive probabilities. There is one caveat, however. At the price $\bar{p}$ in Fig. 1, the uninformed agents no longer conclude *with certainty* that the market has not received any information. Instead, they acknowledge that this price could also have been generated by the signal $\hat{\epsilon}$, and they attach a positive probability to that eventuality. The smaller is the prior probability of $\hat{\epsilon}$ the more inclined are outsiders to believe that the price $\bar{p}$ has been generated by the absence of any new information rather than by the signal $\hat{\epsilon}$.

Thus it seems, that the basic informational mechanism which causes rational over- and undervaluation of assets in this paper is quite general and might be even more pervasive in more elaborate market and information structures. We hope that our study will be one step in advancing the debate about the efficient market hypothesis by demonstrating that informational problems may distort the price system even in fully rational and competitive economies.

Appendix

In this appendix we prove Proposition 1.
Define

$$A := \frac{1}{\lambda b + (1-\lambda)\bar{b}}, \qquad B := -a\sigma_y^2 A$$

$$\bar{A} := \frac{\lambda(\sigma_y^2 + \sigma_\epsilon^2)}{\lambda b(\sigma_y^2 + \sigma_\epsilon^2) + (1-\lambda)\bar{b}\sigma_y^2}, \qquad \bar{B} := \frac{(1-\lambda)\sigma_y^2 \mu_\epsilon - a\sigma_y^2(\sigma_y^2 + \sigma_\epsilon^2)}{\lambda b(\sigma_y^2 + \sigma_\epsilon^2) + (1-\lambda)\bar{b}\sigma_y^2}$$

i) To prove that $(\{\bar{p}\}, \varphi)$ is an equilibrium we have to show that the asset market clears.

   (a) at the price $\bar{p}$, if no signal has been observed;
   (b) at any price $p \in \varphi(\epsilon)$, if the signal ϵ has been observed.
      (a) If no signal has been observed, there is nothing to be learned from the asset price, and hence the asset demands of insiders and outsiders are given by (11) and (13). Inserting $\bar{p}$ yields $\lambda x(\bar{p}) + (1-\lambda)\bar{x}(\bar{p}) = 0$, thus the asset market clears.
      (b) Note from Fig. 1 that any price $p \in \varphi(\epsilon)$, $\hat{\hat{\epsilon}} \neq \epsilon \neq \hat{\epsilon}$, reveals the information signal ϵ through $\epsilon = A^{-1}(p - B)$. Thus, given any such price, all agents are informed about ϵ, and consequently their asset demands are given by (10) and (12), respectively. Inserting $p = A\epsilon + B$, we get $\lambda x(p) + (1-\lambda)\bar{x}(p) = 0$, i.e., asset market equilibrium.
      If $\epsilon = \hat{\epsilon}$, the price $\hat{p} := A\hat{\epsilon} + B \in \varphi(\hat{\epsilon})$ again reveals the information. The same procedure as above shows that $\hat{p}$ is an equilibrium price. It remains to show that $\bar{p} \in \varphi(\hat{\epsilon})$ also clears the market if the signal $\hat{\epsilon}$ has been observed. Since the outsiders know the mechanism according to which prices are formed on the market, they conclude from observing $\bar{p}$ that either the signal $\hat{\epsilon}$ has realized or the market has not received any information at all (see Fig. 1). The ex ante probability for the signal $\hat{\epsilon}$ is zero since ϵ is drawn from a continuous distribution. The ex ante probability for the event that no signal has been observed is α, hence positive. Thus, conditional on the information conveyed by the price $\bar{p}$, the updated probabilities for the signal $\hat{\epsilon}$ and for the non-information event are 0 and 1, respectively. The outsiders therefore rationally conclude that there is no insider information on the market and formulate their asset demands according to (13). The asset demand of an insider is given by (10). Using $\bar{p}$ in (10) and (13) one easily verifies that the aggregate excess demand for assets vanishes.
      Finally, if $\epsilon = \hat{\hat{\epsilon}}$, then market clearing will not be achieved at any price. The 'efficient price' $\bar{p} = A\hat{\hat{\epsilon}} + B$ would clear the market, if *all* agents

base their investment decisions on the information $\hat{\hat{\epsilon}}$. Yet, as we have seen above, the price $\bar{p}$ leads outsiders to believe that the market has not received any insider information. Any other price $p \neq \bar{p}$ reveals the information. With all agents informed, however, $p = \bar{p}$ would be the unique market clearing price.

ii) If $b = \bar{b}$, then it is easy to show that $\hat{\hat{\epsilon}} = \hat{\epsilon}$ and $\bar{p} = \hat{p}$ holds. In this case the equilibrium price correspondence found in part (i) of this proof obviously simplifies to $\varphi(\epsilon) = \{A\epsilon + B\}$. □

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